LARGE DEVIATIONS FOR SOLUTIONS TO STOCHASTIC RECURRENCE EQUATIONS IN RISK THEORY

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Abstract. In this paper we consider the stochastic recurrence equation $Y_t = A_t\, Y_{t-1} + B_t$ for an iid sequence of pairs (A_t, B_t) of non-negative random variables, where we assume that B_t is regularly varying with index $\kappa > 0$ and $EA_t^\kappa < 1$. We show that the stationary solution (Y_t) to this equation has regularly varying finite-dimensional distributions with index κ. This implies that the partial sums $S_n = Y_1 + \cdots + Y_n$ of this process are regularly varying. For $\kappa > 1$, we also study the large deviation properties $P(S_n - ES_n > x)\,, \quad x \geq x_n$, for some sequence $x_n \to \infty$) whose growth depends on the heaviness of the tail of the distribution of Y_1. We show that the relation $P(S_n - ES_n > x) \sim c\, n\, P(Y_1 > x)$ holds uniformly for $x \geq x_n$ and some constant $c > 0$.

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